Sufficient Grounds

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1 Introduction

Recent years have seen a growing interest in *grounding*. In a nutshell, grounding is thought to relate true propositions such that the truth that ϕ grounds the truth that ψ, if the proposition that ϕ (in symbols: $|\phi|$) is the metaphysical explanation as to why ψ. For example [Fine, 2001, p. 15],

> The truth that Jack and Jill are married is grounded in the truth that Jack is married to Jill.

In the following, I will examine whether in such cases of grounding, the truth that ϕ also is *sufficient* for the truth of $|\psi|$. In other words, I intend to clarify the connections between grounding and sufficiency.

GS If $|\phi|$ grounds $|\psi|$ then $|\phi|$ is sufficient for $|\psi|$.

SG If $|\phi|$ is sufficient for $|\psi|$ then $|\phi|$ grounds $|\psi|$.

2 Principles of Grounding

To identify its relation to sufficiency, I first need to say a bit more about the grounding relation. Most authors take it to be a primitive notion [Fine, 2001, Batchelor, 2010, Rosen, 2010]. Therefore, I will also not attempt a definition. Instead, I describe grounding by a small number of principles.

First, the grounds of the truth that ϕ explain why ϕ.

E If x grounds y then x accounts for y.

Since no truth is an explanation of itself, this close link between grounding and explanation implies the relation to be *irreflexive*. Although consequence of the explanatory character of grounding, it will be useful to have
this feature displayed for future reference.

I $\neg\exists(x \text{ grounds } x)$

Second, the grounding relation is transitive.

T If x grounds y and y grounds z then x grounds z.

In the relevant literature this is not always assumed; some authors prefer a relation of immediate grounding [Batchelor, 2010]. In these cases, too, however, a transitive notion is obtained as the ancestral relation.

Notice that transitivity and irreflexivity together make grounding an asymmetric relation.

A If x grounds y then y does not ground x.

Grounding need not be well-founded. The general notion I have in mind allows for infinitely descending chains as long as they do not end up in cycles.

Grounding is linked to logic by the following principles [Fine, 2010, Correia, 2011].

G$\vee$ If ϕ then $|\phi|$ grounds $|\phi \vee \psi|$ (as does the truth that ψ).

G$\neg$ If ϕ, then $|\phi|$ grounds the truth that $\neg\neg\phi$

G$\forall$ If $\phi(t)$ then $\phi(t)$ partially grounds $\forall x \phi(x)$.

These principles of grounding can be seen as reflecting the classical truth conditions. However, notice that only true propositions are assumed to ground their logical functions.

The present notion of grounding has a close cognate: dependence. Some authors simply take one to be the inverse of the other [Rosen, 2010], such that x grounds y just in case y depends on x. For others again, dependence is the (inverse) first order analogue of grounding. Whereas grounding relates *propositions*, it is *objects* that depend on one another (Audi 2010, Fine).

2.0.1 The Distinction between Complete and Partial Grounding

One basic question about grounding is not yet answered. If x grounds y, is x the one and only ground for y, or just one among a number of grounds? Call the first notion 'complete' and the second 'partial' grounding.

In the literature, authors usually understand grounding in one of these ways, and define 'complete' in terms of partial grounding, or vice versa. The reason why I have left it open is that the distinction between complete and partial grounds bears on how grounding relates to sufficiency.

In fact, I do not see how the two understandings can be spelt out without evoking the notion of sufficiency. Here is the natural first shot. $>$ $|\phi|$ is a *complete* ground of $|\psi|$ if $|\phi|$ grounds $|\psi|$ and there's no other truth that $|\psi|$ is grounded in.
However, this way of spelling out complete grounding does *not* work in combination with transitivity. I want to allow for cases of complete grounding where x totally grounds y and y totally grounds z. The transitivity of grounding, however, implies that both x and y are complete grounds of z, which contradicts the purported definition.

This outcome is avoided if one resorts to the relation of *immediate* grounding.

> x is the complete ground for y if x is the only immediate ground for y. Otherwise, x *partially* grounds y.

However, this characterization does not satisfy, either. It implies that only immediate grounds may be complete. This leads to wrong predictions. Consider

TS It's true that it's truth that snow is white.

This truth is grounded in |'Snow is white'|. In fact, it is *completely* grounded in this proposition. Since |'Snow is white'|, however, is a merely mediate ground, the above distinction between complete and partial grounds cannot accommodate this pre-theoretic certainty. Fortunately, the same considerations lead us on the right track. Why is TS completely grounded in |'Snow is white'|? Intuitively, all there is to make TS true is that snow is white. In other words, this mundane truth suffices for the truth of TS.

We are therefore led to distinguish between complete and partial grounds in terms of *sufficiency.*

CP 1. x *completely* grounds y iff (i) x grounds y and (ii) x sufficient for y. 2. x *partially* grounds y if (i) x grounds y and (ii) x *not* sufficient for y.

I presume that this is in fact the only adequate way of drawing the distinction. If so, then we have found a close connection between the notion of grounding and that of sufficiency. How does it bear on the grounding-sufficiency links from above, the principles SG and GS? First consider

GS. If $|\phi|$ grounds $|\psi|$ then $|\phi|$ is sufficient for $|\psi|$.

It is easily seen that this assumption rules out partial grounds as characterized by CP. If any ground for x already suffices for x, then the clause 2(ii) is never satisfied, and partial grounding an empty category. Intuitively, though, there seem to be both complete and partial grounds. Hence, the definition CP is only adequate if GS does not hold.

Second, according to CP, any sufficient ground is a complete ground. Thus, it settles the question of SG ("if sufficient, then ground") to some extent. SG holds for complete grounds.

ScG If $|\phi|$ is sufficient for $|\psi|$ then $|\phi|$ completely grounds $|\psi|$.

This may well be found discomforting. We would not want such a close link between two independent notions to be decided, if only partially, by mere definition. The distinction between complete and partial grounds would look better if ScG was motivated from reflection on sufficiency and grounding.

More severely even, if independent considerations show ScG to fail, CP could not be upheld, either. Thus, a basic question about grounding, the distinction between partial and total grounds, has been shown to rely on the connection between grounding and sufficiency. Unfortunately, how they relate is not obvious since sufficiency itself is a pre-theoretic notion that can be specified in various ways.

In the next section, I will therefore set out to consider different understandings of sufficiency, and against this background evaluate the sufficiency-grounding links SG and GS.

3 Implication

The simplest way of understanding sufficiency is in terms of implication. Thus, to say that the truth that ϕ suffices for the truth that ψ, is to say that if ϕ then ψ.

On this basis, grounding-sufficiency links become

GS$\rightarrow$ If $[\phi]$ grounds $[\psi]$ then, if ϕ then ψ.

SG$\to$ $[\phi]$ grounds $[\psi]$ only if, if ϕ then ψ.

SG$\to$ is clearly false. Any proposition implies itself, whereas grounding is irreflexive.

GS$\to$ may seem more promising. R. Batchelor recently argued that cases of grounding are cases of implication [Batchelor, 2010]. However, this holds only against the background of his Tractarian fact theory, with its independent atomic situations. Beyond that, counterexamples are easily found. Indeed, grounding statements are especially interesting in non-Tractarian settings. Just consider the proposition that the ball is maroon, which does not imply, but grounds |'The ball is red'|. Therefore, GS$\to$ does not hold in full generality.

I conclude that the implication reading of sufficiency does not support either sufficiency-grounding link.

If we understand sufficiency this way, we may distinguish between complete and partial grounding as proposed above (CP). The failure of GS$\to$ allows for insufficient, partial grounds. On the other hand, although SG$\to$ fails, some grounds may well be sufficient. *Complete* grounding in the sense of CP becomes what we would intuitively like it to be, an interesting, special kind of grounding.

4 Determination

Understanding '$|\phi|$ suffices for $|\psi|$' 'The truth of the proposition that ϕ suffices to make true the proposition that ψ' as '$\phi \to \psi$' is to view sufficiency through the lens of *logic*. This reading does not support a strong link between sufficiency and grounding. On reflection, this negative result does not surprise: grounding is a concept from metaphysics.

One way of understanding sufficiency as a metaphysical notion is the following. We understand sufficiency statements as a kind of implication, but require truth preservation only for metaphysically possible worlds. That is, we set

MI The truth that ϕ suffices for the truth that ψ if $\forall w(w$ metaphysically possible $\to (w \vDash \phi \to w \vDash \psi))$

Sufficiency thus becomes a notion of *metaphysical* implication. However, it does not support the sufficiency-grounding link SG, either, because we have again arrived at a *reflexive* notion of sufficiency. The reason is that the inner conditional on the right-hand side of MI is true if its antecedent is just its consequent (ϕ is true at any world w only if ϕ is true at w). Speaking loosely, there is still too much implication in the notion.

In fact, MI fails to express an important aspect of the notion of metaphysical sufficiency. In the end, the truth that ϕ may be sufficient for the truth that ψ without being *necessary* for it. In other words, it is possible that ψ without it being the case that ϕ. Making this explicit, we arrive at the following understanding of metaphysical sufficiency.

MS The truth that ϕ suffices for the truth that ψ if $\Box(\phi \to \psi) \wedge \Diamond(\psi \wedge \neq \phi)$.

This notion of sufficiency coincides with Stephen Yablo's (1992) analysis of determination [Yablo, 1992, Def. Δ]. And 'determines' seems to me indeed an apt expression of the metaphysical sense of 'suffices': the truth that ϕ suffices for the truth that ψ if the former determines the world to be such that ψ.[1]

Since necessarily, ϕ or $\neg\phi$, the relation of sufficiency defined by MS is irreflexive. This makes it an attractive interpretation of the bridge principles GS and SG.

However, the relation defined by MS is not transitive. In S5, the second conjunct of its right-hand side, say $\Diamond(\chi \wedge \neg\phi)$, may not hold even if we have $\Diamond(\psi \wedge \neg\phi)$ and $\Diamond(\chi \wedge \neg\psi)$.[2] Therefore, sufficiency in the sense of MS will not validate the principle GS, either.

It may be thought that this difficulty could be avoided if the underlying modal logic was strengthened. However, the target is *metaphysical* sufficiency, and for good reasons it is widely held that S5 captures metaphysical modality. Therefore, S5 is the only relevant modal logic. Moreover, the required inference from $\Diamond(\psi \wedge \phi)$ to $\Diamond\psi \wedge \Diamond\phi)$ is not valid on any reasonable understanding of possibility. For, surely Greece could have defaulted already in 2010 ($\Diamond\phi$). It has not, though ($\neg\phi$, hence $\Diamond\neg\phi$). But it has never been possible for it to default and *not* default at the same time ($\neg\Diamond(\phi \wedge \neg\phi)$).

Maybe, I have been on the wrong track anyway. I have assumed that the sufficiency of x for y may be understood not in terms of implication, but as a stricter relation, which is paraphrased as 'x makes it the case that y'. However, this is just how grounding is

[1] Notice, however, that the relevant sense of determination is more general than that of the determinate-determinable relation. The truth that Socrates is human determines, in the present sense, the truth that he is an animal. But humanity does not determine the determinable of being an animal, since the former can be defined in terms of the latter [Searle, 1967].

[2] Consider the model

$$\begin{tikzcd} \psi \wedge \neg\phi & w_1 \arrow[rr, leftrightarrow] \arrow[dr, leftrightarrow] & & w_2 & \chi \wedge \neg\psi \\ & & @ \arrow[ur, leftrightarrow] & & \end{tikzcd} \quad (1)$$

circumscribed in the literature [Fine, 2001, Audi, , Fine, 2010]. And it seems to be in the spirit of other intuitive characterizations, such as

- Its being the case that ψ *consists in nothing more* than its being the case that ϕ [Fine, 2001, p. 15].
- $|\psi|$ is true in virtue of the truth that ϕ [Rosen, 2010, Correia, 2011].
- The truth that ϕ carries with it the truth of $|\phi|$ [Batchelor, 2010].

I assume that these formulations, although informal, single out a definite concept. Further, I think that they characterize sufficiency just as well as grounding. If so, then we have good reason to think that there is a natural understanding of sufficiency as grounding. On this reading both SG and GS hold trivially.

Consequently, the proposed distinction between complete and partial grounding (p. 3) must not be understood in terms of this sufficiency notion. Since every ground is sufficient (GS) the definition of partial grounding is empty, and fails to characterize the intuitive notion.

5 Conclusion

I conclude that on one hand, if sufficiency is understood in terms of implication, grounding and sufficiency fall apart.

However, sufficiency may also be understood as a stricter relation of determination, or *making it the case*. Arguably, this alternative characterization may be taken as just a different way of expressing groundedness.

These different understandings of sufficiency, and their connection with grounding, bears on the distinction between complete and partial grounds. Sufficiency-as-implication allows us to draw this distinction in terms of sufficiency (CP) (p. 3). Sufficiency-as-grounding, though, rules out this definition of 'complete' and 'partial' grounding in terms of sufficiency.

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