Groundedness and Applications

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1 Introduction

- Sets
  - type theory
  - ZFC
- Truth
  - Tarski
  - Kripke: stronger, as legitimate
- A general principle at work in both cases: Groundedness
- A grounded theory is safe even if it violates the Vicious Circle principle, because it is safely obtained from some ground

2 History

2.1 Roots

- Conjecture: both predicativity and well-foundedness have fed into the idea of groundedness.
- Russell's VCP
- Type theory
  - Limitations

- ZF axiomatic set theory
- Justification?
- Iterative conception
  - Stage theory: The statements of ZF are justified
- Question: Which role, if any, did constructivism play in the development of groundedness?

2.2 Turn to Semantic Groundedness

- Herzberger 1970: The justification of ZF from well-foundedness as paradigm for the treatment of the semantic paradoxes.
- Kripke 1975 provides new, independent motivation for semantic groundedness from speaker-scenario, and a formal definition in terms of an operator, the famous Kripke-jump.

2.3 Discussion and Elaboration of Kripke

- Yablo 1982
- McCarthy 1988
- Gaiffman 1992
- Leitgeb 2005

3 Philosophical conception

This section is the core of my thesis. In it, I aim to develop a general notion of groundedness. My method here is philosophical in the sense that I

- accommodate pre-theoretic ideas,
- address philosophical questions and worries
- use simple, non-technical terminology,

3.1 Essential Features of Groundedness

- Starting point: S grounded in G iff S is obtained from G by iterated application of some grounding operation γ.

3.1.1 Iteration

- finite or transfinite?

3.1.2 Grounding

- No detours
- Cumulation?

3.2 Delineating the Conception

- How does my account relate to groundedness ideas as are discussed in metaphysics [Liggins, 2008, Rosen, 2010]?
  - truthmakers
  - ontological dependence

3.2.1 Epistemology

- Groundedness is often motivated in epistemic terms (justification, Kripke: Speaker learning truth, Fine: Gabriel getting to know proper classes)
- But this is mere heuristics - groundedness is not an epistemic notion.
- Question: How would epistemic groundedness look like? Difference from justification?

3.2.2 Reduction

- Does S being grounded in G allow S to be reduced to G?
  - In which sense?
  - Under which conditions?

3.2.3 Modality

- Does calling S grounded in G commit to actualism about S?
  - Is there merely potential groundedness?
  - Apply distinction between potential and actual groundedness to paradigm cases.

3.3 Implications

3.3.1 Realism, anti-realism

- How far does the constructive and temporal terminology carry in which groundedness is usually described?
- Is the general conception indifferent towards the ontological status of grounded objects?
  - If not, can the realist make use of it? Under which conditions?
- Does ungroundedness imply inconsistency? If a theory is ungrounded, is this sufficient reason to reject it, or only if one has identified an independent problem?

4 Formal theory

Having developed a philosophical conception in the preceding section I now turn to supplement it by a formal definition of groundedness. This definition is supposed to

- be co-extensive with the philosophical conception
  - Thus, it allows me to *prove* groundedness as well as ungroundedness
- accommodate central features of the informal account
  - iteration
  - grounding operation
- satisfy, if possible, criteria of economy and elegance.

4.1 Iteration

4.2 Grounding

- Explicit definition or Axioms?

4.3 Delineating

- The formal definition of groundedness is not just a different way of describing inductive definitions.
  - Being a fixed point not necessary
  - and not sufficient either: examples of ungrounded fixed point.

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5 Applications

- Are any of the following grounded, if so in what, and how useful is this for the respective philosophical discussion?

5.1 Abstraction principles

- [Linnebo, 2009]

5.2 Large cardinals

- Depends on how I spell out the iteration aspect of groundedness.

5.3 Forcing extensions

- Motivation: Analogy between forcing in arithmetic and the Kripke construction.
- Forcing in set theory
- Relate to 'set-theoretic geology' (J. Hamkins, G. Fuchs)

5.4 NF

5.5 Hyper-Sets

- ZFC$^-$+AFA [Aczel, 1988]
- Does Incurvati's graph conception point at how to ground hypersets?

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