On the Philosophy of Semantic Groundedness*

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1 Introduction

The present paper connects two separate strands of research. On the one hand, recent years have seen a growing interest in statements such as

> The fact that 'plus' means addition is grounded in facts about our language community.

Some philosophers have argued for grounding as sui generis metaphysical explanation (Fine 2001; Correia 2005; Rosen 2010). Others again have dismissed this notion as unintelligible (Hofweber 2009; Daly 2011).

In logic, on the other hand, formal theories of truth (Herzberger 1970,kr1975,ya1982,,) have often been motivated from the thought that

> semantic truth is grounded in truth of non-semantic sentences.[1]

Thus, philosophers in two quite separate areas have used the word 'grounding'. This may just be coincidence – or, there is in fact a deeper connection yet to be elucidated. In this paper, I will propose to understand semantic groundedness in terms of the metaphysical grounding notion.

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[1]For example, this thought underlies the work of Herzberger (1970); Kripke (1975); Yablo (1982); McCarthy (1988); Leitgeb (2005).

This interpretation is of interest to both logic and metaphysics. On the one hand, it provides the metaphysicist with evidence not only that the concept is intelligible, but that its application leads to substantial results beyond metaphysics.

On the other hand, an interpretation of semantic groundedness in terms of metaphysical grounding strengthens the case for grounded truth theory. This will be the focus of the present paper.

It is structured as follows. First (§2), I will argue that as a response to paradox, semantic groundedness must account for the truth principles that are endorsed. In a slogan: the grounds of truth must explain truth. For this, the formal concepts of groundedness need to be supplemented with philosophical content. I consider different interpretations of semantic groundedness, none of which make the grounds of truth explanations in a sufficiently strong sense. Metaphysical grounding, however, provides just this strong sense of explanation.

Having made this proposal, I will have to do two things. First, I have to specify what it means for semantic groundedness to ensure metaphysical groundedness. As a first step towards this, I will use Kit Fine's *logic of ground* Fine (2011) to formulate at least necessary conditions on a relation to behave like metaphysical grounding (§ 3.1). Second, I have to show that prominent definitions of semantic groundedness allow for this interpretation. As it will turn out (§ 4), this is a non-trivial task. However, I will give a general method that allows to obtain, given a standard definition of semantic groundedness, a grounding relation that obeys Fine's logic (§ 5).

2 Semantic Groundedness and Philosophical Explanation

In this section, I will argue that the friend of grounded truth must go beyond her formal concept of groundedness and supplement it with a notion of philosophical explanation.

2.1 The Need for Explanation

What principles should we use to reason about truth? The threat of paradox requires careful selection. To keep things simple, let us assume that it is *sentences* to which we assign truth or falsity.

Intuitively, we call a sentence φ true just in case φ. This principle is captured by Tarski's schema (T). But this principle is inconsistent with the fact that there is a sentence λ equivalent to the sentence 'λ is not true'.

Thus, we find ourselves in a paradox. We have independent reason to believe principles which prove jointly incompatible. We need to revise our principles of truth.

This challenge has been taken up by logician-philosophers who, over the past decades, have created an array of formal theories of truth.[2]

Any response to paradox, however, must include a philosophical account of the principles endorsed. This can be seen as follows. By the nature of paradox, all assumptions involved are initially plausible. Unless the philosopher is willing to declare bankrupt she needs to replace these seemings by better explanation.

One particularly fruitful approach was initiated by Herzberger (1970). To explain why certain pathological sentences lead to paradox, he formulated a general condition:

> Any given sentence determines a statement only if it is grounded or is non-semantic. (Herzberger 1970, 149)

For a sentence to determine a statement is for it to be either true or false (i.e. satisfy Excluded Middle, i.e. have a *classical* truth value) (Herzberger 1970, 6). Hence, Herzberger's grounding condition implies:

(G1) Any semantic sentence ϕ is true or false only if it is grounded or nonsemantic.

Before I turn to my main argument that semantic groundedness needs supplementation with a notion of explanation, let me first clarify the notion of semantic groundedness. I focus on the standard case, where the language of arithmetic $\mathcal{L}$ is extended by a predicate 'T', to be read 'is true'.

What is it for '$T\ulcorner 0 < 1 \urcorner$' to be grounded in '0<1'? Various accounts have been offered (Kripke 1975; Yablo 1982; Maudlin 2004; Leitgeb 2005). All of these, however, are meant to capture the same pre-theoretic notion. For the time being, my interest is in this underlying idea of grounding.

I assume that it is best formulated in terms of an intuitive notion of what a sentence is *about*. Following Herzberger (1970, 147f), we can then speak of the *domain* of a sentence φ as the set of everything that φ is about. The pre-theoretic notion of groundedness then becomes the following: To call a sentence φ grounded is to say that the relation of being about is well-founded on the domain of φ. In a slogan: φ is grounded just in case tracing what it is about terminates.

[2]For an up-to-date selection of examples consult Field (2008),Halbach (2011) or Horsten (2011).

$$\forall x(\underset{\cdot}{Even}(x) \to Tx)) \tag{1}$$

For example, the sentence (1) is grounded if aboutness is well-founded on the domain of (1). At a closer look, however, we find that this cannot be all there is to say about the groundedness of (1). There are different, mutually incompatible ways of fixing the domain of (1). In one sense, (1) is about the even numbers. If we take this to be the domain of (1), its groundedness becomes trivial, as 2, 4 and the others are not about anything. In another sense, however, (1) is about the sentences *encoded* by even numbers. On this reading, the groundedness of (1) is not trivial at all. We need to check carefully, which sentences are encoded by even numbers. These may include the sentence (1) itself in which case this sentence is ungrounded, or they may all be nonsemantic, in which case (1) is grounded. Thus, whichever way we fix the domain of (1), something different is said by "(1) is grounded".

Of course, when philosophers discuss semantic groundedness, it is clear how we should understand the domain of a given sentence. In the present example, it is the latter domain that we are interested in, i.e. the sentences encoded by even numbers. This is so, because we are interested in the truth of the sentences of our extended language, and the sentence (1), on its intended interpretation, says that certain sentences are true. Generally, when discussing semantic groundedness, not just any way of fixing the domain of a sentence φ is relevant. The domain that we are interested in collects those sentences which φ says are true.

This assumption is implicit in the literature of semantic groundedness. On its basis, the content of statements such as "φ is grounded" can be specified further. If we fix the domain of a sentence φ by looking at which sentences occur in φ within the scope of 'T', then it is all and only the sentences of the nonsemantic base language $\mathcal{L}$ that are not about any other sentences. Since we take φ to be grounded just in case the relation of being about *in the sense relevant for truth theory* is well-founded on the domain of φ, this means that φ is grounded just in case every chain of being about (in the present sense) terminates in a nonsemantic sentence. This should appear obvious. Of course, the groundedness of a sentence is a matter of how it relates to the sentences of the base language. To makes this explicit, instead of groundedness I will speak of semantic *grounding* as a relation between a sentence containing 'T' and some sentences of the nonsemantic base language.

2.1.1 Logical form of Semantic Grounding

Let me now turn to the logical form of semantic grounding. Herzberger's quote above suggests that it is *sentences* which are grounded. Against this appearance, I will argue that the relata of semantic grounding are meta-theoretic propositions of the form 'φ is true in M', for a sentence φ and a model M.

On the present intuitive account, whether a sentence φ is grounded or not is a matter of what φ is about. Of course, the sentence by itself is not about anything. Only if we interpret the string of characters we are able to "trace down what it is about". Hence, the groundedness of φ can only be determined with respect to a *model.* Thus, it is not φ that we should call 'grounded' but the meta-theoretic fact that what is said by the sentence φ as interpreted in a model M holds in M, i.e. that φ is true in M.

For similar reasons, it is also not the sentences of the base language $\mathcal{L}$ which φ is grounded in, rather the fact that these sentences have the truth value that they have, as interpreted in the base model. I therefore propose to understand semantic grounding as a many-one relation between facts of the form 'φ is true in M'. Let me abbreviate these facts by sentence-model pairs. For example, to say that "0<1' is true' is grounded is to say that ("0<1' is true',M) is grounded.[3]

Thus, Herzberger's *grounding condition* becomes the following claim.

(G2) Any semantic sentence ϕ is true or false in M only if there is a set of sentences Ψ of the base language $\mathcal{L}$ such that (ϕ, M) is grounded in $\{(\psi, N) : \psi \in \Psi\}$, where N is the standard model of arithmetic.

Although Herzberger formulates a necessary condition, it is also sufficient. Again, this fact does not rely on how groundedness is defined formally, but is implicit in the pre-theoretic understanding. This can be seen as follows.

Assume that the truth of φ in M is grounded. Thus, the relation of being-about, however we wish to specify it, is well-founded on the domain of φ in M. We may identify the *rank* of (φ,M) with the length of the longest aboutness-chain that stretches from (φ,M) down to truth in the base model.

Since being-about is well-founded on the domain, we can reason by induction on the rank of (φ,M). In the base case, when the rank of (φ,M) is 1, φ in M is directly

[3]In most cases, M would be the standard model of arithmetic $\mathbb{N}$ expanded by some predicate T to interpret 'is true' ('$\mathbb{N}(T)$').

about nonsemantic sentences $\psi_0, \psi_1, \ldots$, possibly infinitely many. I assumed, in line with Herzberger and all cases considered below, that the language of φ extends the base language $\mathcal{L}$ by just the predicate symbol 'T', to be read as '... is true'. Therefore, I may assume that φ in M states the *truth* of $\psi_0, \psi_1, \ldots$. Since these sentences are nonsemantic, they are either true or false in the base model. Hence, ϕ is also ensured to obey Excluded Middle in M.

At the induction step, assume that (φ,M) states the truth of $\psi_0, \psi_1, \ldots$. Each is either true or false, although it may contain 'T'. Hence, ϕ is again ensured to have a classical truth value in M. Consequently, any grounded sentence is either true or false. Groundedness is not only necessary but also sufficient for being true or false.

(G3) Any semantic sentence ϕ is true or false in M if and only if there is a set of sentences Ψ of the base language $\mathcal{L}$ such that (ϕ, M) is grounded in $\{(\psi, N) : \psi \in \Psi\}$, where N is the standard model of arithmetic.

Having thus put the pre-theoretic notion of semantic groundedness in a workable form, I now turn to my main argument. I will argue that semantic groundedness is in need of philosophical enrichment.

Paradox tells us that the Tarski schema needs restriction. However, once we give up the simple principle (T), we face an embarrassment of riches. a multitude of As Van McGee has shown (1992), there are infinitely many maximally consistent but jointly incompatible restrictions of the T-schema. If we want to reason coherently about truth, we need to decide for some such set. In particular, assigning a truth value to a sentence containing the truth predicate amounts to somehow restricting (T).

Herzberger's groundedness condition, and my development of it (G3), allows us to assign a truth value to a certain class of sentences. Thus, semantic groundedness gives rise to a certain restriction of Tarski's truth schema. An arbitrary choice, however, would be unacceptable. Consequently, if we want to endorse a particular system of reasoning about truth we need a philosophical reason for this choice. Hence, we can assign a truth value to ϕ in a model M only if ϕ's truth in M is accounted for. Therefore, (G3) implies that if (φ,M) is grounded, then its truth is accounted for.

However, not any account would suffice. The paradoxes challenge basic patterns of reasoning. In order to assure ourselves that we have chosen the right set of truth-theoretic principles, we therefore must provide an explanation that withstands philosophical scrutiny. Thus, the philosophical challenge of paradox requires our truth theory to be accounted for in a strong sense.

At the very least, we need to be sure that our account does not mislead us. Nothing can be accounted for in this strong sense without really being the case. In other words, in order to answer the challenge of paradox we need *factive* explanations.[4]

Consequently, the truth of φ in M is accounted for in the strong sense needed, only if φ is true in M. Thus, if we uphold semantic groundedness as a necessary condition on truth, then the truth of φ in M is accounted for only if there are sentences of the base language in whose truth or falsity (φ,M) is grounded. In sum, the groundedness approach to truth theory answers the philosophical challenge of paradox only if the following correlation between semantic groundedness and explanation holds.

(G4) For any semantic sentence ϕ, (ϕ, M) is grounded in $\{(\psi, N) : \psi \in \Psi\}$ just in case the fact that ϕ is true in M is accounted for by the fact that ψ_0 is true in N, ψ_1 is true in N and so on.

For example, in Kripke's truth theory (1975) the liar sentence is denied truth value not by stipulation, but because its truth is not grounded in arithmetical truth. Whichever truth value we ascribed to it, it would not be accounted for by arithmetical truth. I take this to be the philosophical appeal of Kripke's proposal.

However, why should we accept (G4)? It is not self-evident that groundedness should ensure philosophical explanation. So far, it has only been defined mathematically, for instance as the least fixed point of the strong Kleene jump operator. For groundedness to provide a philosophical account, such formal definitions do not suffice. They need to be enriched philosophically. Below, I will consider different ways of doing so and argue that none of them validates the principle (G4).

2.2 Leitgeb's Formal Concept of Semantic Groundedness

For this purpose, I need to be more specific than I have been so far. I need to discuss the formalized concepts of semantic groundedness.

I focus on the most recent example of grounded truth: Hannes Leitgeb's (2005). Leitgeb works within the usual setting of formal truth theory. To the language of arithmetic

[4] I would ask the reader to keep this in mind, the factivity of explanation will play an important role below.

$\mathcal{L}$ is added a monadic predicate 'T', to be read 'is true'. The extension of this predicate is a set of sentences of the extended language $\mathcal{L}_t$. Whichever set is taken for this, a different model of $\mathcal{L}_t$ is obtained. The idea behind Leitgeb's notion of groundedness is that the truth value of sentences containing 'T' (in symbols: '$Val_Y(\phi)$' for some sentence ϕ and a set of sentences Y) *depends* on this choice.[5]

Definition 1. The sentence ϕ depends on the set of sentences X if variation in truth value requires variation of the interpretation of 'T' with respect to X: for all sets Y, Z, $Val_Y(\phi) \neq Val_Z(\phi)$ only if $Val_{Y \cap X}(\phi) \neq Val_{Z \cap X}(\phi)$.

Thus, a sentence ϕ is said to depend on a set of sentences X, if it matters to the interpretation of ϕ at most whether or not the X are in the extension of 'T'. For example, '$T\ulcorner 0 < 1 \urcorner$' depends on {'$0 < 1$'}. The sentence can be true in one model and false in another only if the extension of 'T' as interpreted in the first differs from the extension of 'T' in the latter with respect to {'$0 < 1$'}.

Leitgeb defines an operator D^{-1} that maps a set of sentences to the set of just those sentences which depend on the first. D^{-1}'s fixed point Φ_{lf} collects all and only the sentences ϕ that depend on sets of nonsemantic sentences. This is Leitgeb's concept of groundedness: ϕ is grounded just in case $\phi \in \Phi_{lf}$ (Leitgeb 2005).

A sentence φ depends on a candidate interpretation X of 'T' if there is no difference in the truth value of φ without a difference in the interpretation of 'T'. It does not matter, however, which truth value is assigned to φ in the model that interprets 'T' by X. In other words, Leitgeb's dependence relation it does not distinguish between truth and falsehood. Thus, the Φ_α include both true and false sentences. To define grounded *truth*, Leitgeb constructs a set Γ_{lf} such that for every sentences $\phi \in \Phi_{lf}$, $\phi \in \Gamma_{lf}$ just in case $\mathbb{N}(\Gamma_{lf}) \vDash \phi$.

Leitgeb restricts Tarski's T-Schema to Φ_{lf} – this is his response to the paradoxes of truth. He writes:

> What kinds of sentences with truth predicate may be inserted *plausibly* and consistently into the T-scheme? [...] We suggest turning to a class of sentences the truth or falsity of which may be said to [...] *depend on*, the truth or falsity of the sentences of [..., $\mathcal{L}$]. (Leitgeb 2005, 160, my emphases)

[5]For ease of readability, I will use small Greek letters both to use and to mention sentences of the language $\mathcal{L}_t$, just as Leitgeb does.

Thus, he assumes the truth or falsity of $\phi \in \Phi_{lf}$ in the model $\mathbb{N}(\Gamma_{lf})$ to be accounted for by arithmetical truth. But, why should we accept this interpretation of Leitgeb's formal concept of groundedness? Φ_{lf} is defined as the set of sentences that depend on the base language $\mathcal{L}$. For example, '$T\ulcorner 0 < 1\urcorner$' is grounded because it depends on {'0<1'}. The question thus becomes: How does the dependence of '$T\ulcorner 0 < 1\urcorner$' on {'0<1'} make the truth of '0<1' in the base model account for the truth of '$T\ulcorner 0 < 1\urcorner$' in the fixed point model $\mathbb{N}(\Gamma_{lf})$?

2.3 The Philosophical Component of Groundedness

Leitgeb's definition of semantic dependence has the following equivalent (Leitgeb 2005, 161).

Lemma 2. ϕ depends on X just in case $\forall Y,\ Val_Y(\phi) = Val_{Y\cap X}(\phi)$

This may be read: 'X suffices for the truth or falsity of ϕ in any model $\mathbb{N}(Y)$ where $X \subseteq Y$'. For example, if '$T\ulcorner 0 < 1\urcorner$' depends on {'0<1'} then {'0<1'} suffices for the truth of '$T\ulcorner 0 < 1\urcorner$' in Leitgeb's model $\mathbb{N}(\Gamma_{lf})$. Is it in this sense that {'0<1'} accounts for the fact that '$T\ulcorner 0 < 1\urcorner$' is true in $\mathbb{N}(\Gamma_{lf})$?

This option must be considered carefully. Sufficiency itself is a pre-theoretic notion that can be understood in various ways.

The simplest way of understanding sufficiency is in terms of the material conditional. Thus, to say that $\{\varphi\}$ suffices for ψ is to say that ψ if ϕ.

However, if this was how the groundedness concept is to be enriched, grounded truth theory would be in bad shape. Since, if we understand sufficiency in terms of the conditional then any proposition trivially suffices for itself. Hence, arithmetical truth would explain semantic truth only as well as semantic truth explains itself – it does not. The paradoxes tell us that type-free truth is *not* self-explanatory. In other words, semantic grounding must be *irreflexive* in order to play the role that Leitgeb and others assign to it. Therefore, sufficiency as understood in terms of the material conditional cannot provide the philosophical justification for grounded truth theory.

This is not just another infelicitous feature of the *material* conditional. Conditionals of more elaborate semantics, such as the relevant conditionals, likewise validate the general law 'If ϕ then ϕ'. Thus, they cannot provide the required notion of explanation, either.

Another sense in which a set of sentences X may suffice for the truth of a sentence ϕ is if X *entails* ϕ.

But, this does not provide the desired notion of philosophical explanation, either. Equally, '$T\ulcorner 0 < 1\urcorner$' can be inferred from any set extending {0<1}. But, none of these other, bigger sets explain its truth value.

In other words, the required sense of explanation is *non-monotone*. Therefore, even entailment exceeds the concept of philosophical explanation that underlies grounded truth theory. Semantic dependence needs to be understood differently to provide Leitgeb's groundedness with the philosophical motivation required in view of the paradoxes.

As a matter of fact, Leitgeb suggests a philosophical interpretation of his dependence relation.

> ... the notion of dependence which we aim at is a kind of *supervenience* (Leitgeb 2005, 160).

Recall Leitgeb's definition of semantic dependence according to which ϕ depends on X just in case there is no difference in the truth value of ϕ without a difference in the interpretation of 'T' with respect to X. Thus, if we conceive of the different extensions of the base model as worlds, then Leitgeb's definition indeed satisfies the standard account of supervenience as necessary covariation.

This idea has some appeal. Supervenience is a versatile, well examined philosophical concept. If semantic groundedness is understood as supervenience, it clearly is provided with philosophical content – unfortunately, though, not with the right content.

For the supervenience reading of groundedness to validate principle G, any sentence supervenient on some set must be accounted for by that set. Logical truths and falsehoods, however, supervene on anything, since their truth value is the same in all models. So, '$T\ulcorner 0 < 1\urcorner \vee \neg T\ulcorner 0 < 1\urcorner$' supervenes, in just Leitgeb's semantic sense, on {'1<0'}. This set, however, does not explain the truth of the disjunction.

This is a well-known shortcoming of the notion of supervenience. It does not properly distinguish explanatory relations between statements that hold of necessity.

3 Grounding Groundedness

The proponent of grounded truth takes the groundedness of her theory to provide it with philosophical motivation. Thus, she enriches her formal definition with philosophical content. I have asked, what is this additional, non-mathematical aspect of groundedness?

In the case of Leitgeb's theory, what is it about a sentence ϕ depending on a set X that makes X account for the truth of ϕ?

I have found that the notion of philosophical explanation implicit in the groundedness-theorist's case is stronger than supervenience or entailment. On reflection, this should not surprise. Groundedness is supposed to outweigh the intuitive plausibility of naive truth theory. This is a specifically philosophical project. Thus, grounding needs to be explanation of specifically philosophical character.

In recent years, such a notion of *sui generis* philosophical explanation has been discussed in metaphysics. It is usually thought of as a many-one relation between true propositions. Roughly, the idea is that one truth holds in virtue of some other truths, such that the former is fully accounted for by the latter. Remarkably, this concept of explanation is called 'grounding'. For example, a communitarianist about meaning may say that

> the meaning of '+' is grounded in facts about our language community.

For the sake of clarity, I will refer to this metaphysical notion as *m-grounding*, and to the semantic notion as *s-grounding*.

Should we understand the argument for grounded truth in terms of m-grounding, along the following lines?

(G5) (ϕ, M) is s-grounded in $\{(\psi, N) : \psi \in \Psi\}$ just in case the fact that ϕ is true in M is m-grounded in the fact that all sentences in Ψ are true in N.

(G5) is a bridge principle between formal truth theory and metaphysics. As such, it provides semantic groundedness with the required philosophical content: grounded truth would be *fully accounted for* by truth in the base language. Further, m-grounding has been found stronger than both entailment and supervenience. Thus, understanding the semantic notion of groundedness along these lines promises to bear rich fruit.

There are two immediate worries about (G5). First, there is no evidence in the literature on semantic groundedness that would support such a close connection between truth theory and metaphysics. It is unreasonable to assume that truth theorists, when proving their fixed point theorems, are in the business of providing metaphysical explanations. My principle may thus appear to be a simple equivocation.

However, (G5) does not require semantic grounding to be a concept of metaphysical explanation. Instead, it merely claims a one-one correlation between s-groundedness and m-groundedness. More generally, my starting point "semantic grounds are explanations" is not meant as an analysis of semantic grounding but as a *desideratum.* What I offer is that, if we can establish such a correlation between semantic groundedness and metaphysical explanations, then this strengthens the philosophical case for grounded truth.

Second, it may be complained that metaphysical grounding is a vague, or esoteric, or even incoherent notion. Thus, nothing is gained by invoking m-grounding to clarify and enrich the truth-theoretic.

I agree. Of course, my proposal to the truth theorist is only as substantial as the metaphysical notion can be made more precise. Fortunately, over the recent years, a number of principles have been identified that provide at least necessary conditions on a relation to express metaphysical grounding. I will now use these principles to specify my proposal.

In particular, I will use the formal system that Kit Fine has proposed very recently (Fine 2011). For purely metaphysical discussions, less technical presentations may suffice. My goal, however, is to clarify the notion of semantic groundedness. This purpose calls for precision; and Fine's 'pure logic of ground' is the most rigorous presentation available today.

3.1 A Logic of Grounding

Fine sets up a calculus which comprises four concepts of m-grounding. This allows him to accommodate a range of views proposed in the metaphysical literature as well as bring out how these notions interact. One of them, *strict full* m-grounding, will emerge as the appropriate interpretation of semantic grounding.

First, Fine distinguishes between a weak and a strict sense of m-grounding. On the one hand, to say that ϕ, ψ, ... weakly ground χ is to say that for it to be the case that χ is for it to be the case that ϕ, ψ, ... (Fine 2011): In particular, any truth weakly grounds itself: weak m-grounding is reflexive.

Strict grounding, on the other hand, is irreflexive. Adopting a useful metaphor of Fine's, strict grounding moves us '... down in the explanatory hierarchy', where weak grounding has moved us merely 'sideways' (Fine 2011).

Above (p. 9), I have noted that semantic grounding must be irreflexive. Therefore, the concept of m-grounding suitable for an interpretation of s-grounding is *strict.* If we understood (G5) in terms of weak m-grounding, nonsemantic truth would account for semantic truth only as well as semantic truth accounts for itself. This reading of (G5) would not make grounded truth theory respond to the philosophical challenge of paradox.

Second, Fine distinguishes between *full* and *partial* m-grounding. This distinction is made often (Audi 2010,fn2010). Fine draws it in terms of sufficiency: ϕ, ψ, ... fully ground χ just in case ϕ, ψ, ... are sufficient to ground χ. *Partial* grounds ϕ, ψ, ... , on the other hand, merely help grounding χ: there are other ξ, ... such that ϕ, ψ, ξ ... fully ground χ.

Is semantic grounding interpreted best by full, or by partial m-grounding? My goal is to interpret Leitgeb's concept of groundedness in terms of m-grounding. On his definition, a sentence is grounded in just in case it depends on the nonsemantic base language. Leitgeb explicitly describes his semantic dependence as a concept of 'total' dependence (Leitgeb 2005, 160). Therefore, I will focus on full strict m-grounding to specify my proposed interpretation of semantic grounding. Let strict full m-grounding be denoted '<' and governed by the following rules.[6]

$$\text{Cut}\ \frac{X_1 < \phi_1 \qquad \dots \qquad X_n < \phi_n \qquad \{\phi_i : 1 \leqslant i \leqslant n\} < \psi}{\bigcup\limits_{1<i<n} X_i < \psi}$$

$$\text{Non-Circularity}\ \frac{X \cup \{\phi\} < \phi}{\perp}$$

Notice that Non-Circularity makes strict full grounding ('<') *non-monotone* in the following sense.

Lemma 3. It is inconsistent with Fine's rules of grounding to assume that for every X, Y, ϕ,

$$\frac{X < \phi}{X \cup Y < \phi}$$

Thus, if the X are full, strict grounds for ϕ then we cannot in general assume ϕ to be grounded in any extension of X. This should not surprise. Presumably, explanation generally is non-monotone. At any rate, non-monotonicity holds for the strong sense of

[6] In the appendix (p. 20), I show how these two rules allow for the derivation of the rest of Fine's system.

explanation which m-grounding is thought to be. Assume that my being in pain is fully accounted for by the fact that my C-fibres are firing. Then it is not the case that my pain is equally fully explained by the fact that my C-fibres are firing and the fact that 1 plus 1 equals 2.

Fine's logic of m-grounding allows me to specify my interpretation of semantic groundedness. In order for s-grounding to be supplemented with the notion of *sui generis* philosophical explanation, it needs to have the formal properties of full, strict m-grounding. This gives rise to the following, general desideratum for definitions of s-groundedness:

Desideratum — There is a relation $\mathcal{G}$ that obeys the rules of strict full grounding such that for any sentence ϕ whose truth value in some designated model M is s-grounded in the truth or falsity in the base model N of some nonsemantic sentences Ψ:

$$\{\psi : \psi \in \Psi\}\mathcal{G}\phi$$

4 Semantic Dependence and Grounding

In this section, I will examine whether Leitgeb's concept of semantic groundedness allows for an interpretation in terms of m-grounding, along the lines of (G5). The previous section has provided a necessary condition: such a reading requires the relevant s-grounding to obey Fine's rules of full, strict m-grounding.

In Leitgeb, for a sentence ϕ to be grounded in the arithmetical base-language, is for it to depend on a set of sentences of this language. Therefore, I will examine whether semantic dependence, or rather its inverse, can be interpreted as full strict m-grounding. More precisely, my question is whether 'ϕ depends on X' satisfies the rules for $\ulcorner X < \phi \urcorner$.

My answer will be that it does not.

4.1 Cut

First, semantic dependence satisfies cut just in case the following holds.

(1) If ϕ_1 depends on X_1, ϕ_2 depends on X_2, ... and ψ depends on $\{\phi_i : 1 \leqslant i \leqslant n\}$ then ψ depends on $\bigcup_{1 \leqslant i \leqslant n} X_i$.

It is easily shown to fail. The sentence '$T\ulcorner T\ulcorner 0 < 1\urcorner\urcorner$' depends on $\{T\ulcorner 0 < 1\urcorner\}$, but not on {'0 < 1'}, although '$T\ulcorner 0 < 1\urcorner$' depends on {'0 < 1'}. This can be seen as follows.

Recall Leitgeb's definition of dependence (p. 8 above). A sentence ϕ depends on a set X just in case the truth value of ϕ changes only if the extension of 'T' changes with respect to X. In the present case, '$T\ulcorner 0 < 1\urcorner$' may be removed from the extension of 'T' whether or not this includes {'0 < 1'}. It simply does not matter for the truth value of '$T\ulcorner T\ulcorner 0 < 1\urcorner\urcorner$' whether or not '$T$' applies to '0 < 1'. Surely, if 'T' is to express truth in $\mathcal{L}_t$, then its extension must include '$T\ulcorner 0 < 1\urcorner$' just in case it contains {'0 < 1'}. However, when we are in the business of identifying what '$T\ulcorner T\ulcorner 0 < 1\urcorner\urcorner$' depends on, '$T$' must not be treated like a truth predicate. Leitgeb's definition of dependence does not quantify over intended interpretations of 'T' but over all subsets of the domain. This feature of semantic dependence prevents it from satisfying the principle 1.

4.2 Monotonicity and Essential Dependence

Second, semantic dependence does not satisfy the Non-Circularity rule of Fine's logic, either. It will be more instructive, though, to show that it violates non-monotonicity in the sense of proposition 3.

Recall that, if ϕ depends on X then it depends on any extension of X (p. 9 above). Leitgeb's concept of semantic dependence allows for adding redundant sentences to any set that ϕ depends on (Leitgeb 2005, 160). Consequently, Leitgeb's dependence does not show the non-monontonicity of full, strict grounding.

At this point, it needs to be noted that Leitgeb does define a stronger, non-monotone concept of dependence (Leitgeb 2005). ϕ *essentially* depends on the set X if X is the least set that ϕ depends upon. For example, '$T\ulcorner 0 < 1\urcorner$' essentially depends on {'0<1'}.

Definition 4. ϕ *essentially* depends on X iff $X = \cap\{Y : \phi \text{ depends on } Y\}$

By definition, essential dependence satisfies the non-monontonicity property of full, strict grounding.

However, Leitgeb has shown that there are grounded sentences which do not depend essentially on any set (Leitgeb 2005, 170). Hence, not for every grounded sentence ϕ there are arithmetical sentences X such that ϕ depends on X. Essential dependence does not show Leitgeb's concept of groundedness, that of being in the fixed point set Φ_{lf} to fulfil the general desideratum from above.

In order to answer the challenge of paradox, truth theoretic groundedness must provide philosophical explanation. Therefore, I have proposed to interpret truth theoretic groundedness in terms of metaphysical grounding. This interpretation requires the grounded sentences to stand in a relation to the nonsemantic base language which obeys the rules of cut and non-monotonicity.

Leitgeb's dependence relation has none of these formal properties. Has Leitgeb failed to define a theory of grounded truth? I do not think so. In the remainder of this paper I will recover from Leitgeb's definition a relation which does after all satisfy the rules of Fine's logic. Moreover, I will do so in a way that generalizes easily and applies equally to Kripke's concept.

5 A Grounding Relation for Semantic Groundedness

In the remainder of this paper I will show that the two most prominent definitions of semantic groundedness, Kripke's and Leitgeb's, satisfy the desideratum. I will develop, on the basis of work by Stephen Yablo, a general method to identify, for both definitions, a relation $\mathcal{G}$ that obeys the rules of strict full grounding. For the ease of presentation, I focus on Leitgeb's concept. Along the way, I will hint at the straightforward generalization.

Recall that Leitgeb's concept of groundedness is the least fixed point of the sequence (Φ_α), defined in terms of his operator D^{-1}, as follows.

Definition 5.

$$\begin{aligned} \Phi_0 &= \varnothing \\ \Phi_{\alpha+1} &= D^{-1}(\Phi_\alpha) \\ \Phi_\alpha &= \bigcup_{\beta<\alpha}(\Phi_\beta), \text{ for } \beta \text{ limit.} \end{aligned}$$

Notice that every grounded ϕ enters the set at some successor stage of this sequence, which suggests the following natural definition of its *rank*.

Definition 6. (rank) The rank of ϕ is the least α such that $\phi \in \Phi_\alpha$.

$$|\phi| = min\{\alpha : \phi \in \Phi_\alpha\}$$

As all ranks are successor ordinals $\geqslant 1$, we can, for any sentence ϕ, single out all and only those sentences of rank less than $|\phi|$. Consider the relation "... is of higher rank than ...", or formally:

Definition 7. (Δ_l)

$$\phi \Delta_l \psi \quad \text{iff} \quad \psi \in \Phi_{|\phi|-1}$$

Thus, a sentence φ stands in the relation Δ_l to another sentence ψ just in case ψ enters the realm of grounded sentences earlier than φ. Δ_l is not very informative: Any grounded φ stands in this relation to every other grounded sentences of lower rank. For example, '$T\ulcorner 0 < 1\urcorner$' stands in Δ_l to '1+3<7'.

However, purely arithmetical sentences do not stand in the relation Δ_l to anything, since they are of rank 1, and Φ_0 is empty. For arithemtical sentences, there simply are not any sentences of lower rank.

Consider sequences of sentences $\vec{\phi} = \langle \psi_0, \psi_1, \psi_2, \ldots \rangle$ where $\psi_0 = \phi$ and for every $\alpha + 1$, $\psi_\alpha \Delta_l \psi_{\alpha+1}$. Let the αth element of $\vec{\phi}$ be denoted '$\vec{\phi}_\alpha$'. Already in the early 1980s, Stephen Yablo showed the generalization of the following[7]

Lemma 8. (Yablo 1982) If $\phi \in \Phi_{lf}$ then every $\vec{\phi}$ is finite.

On this basis, a relation can be identified which obeys Fine's rules of full, strict m-grounding.

Definition 9. $(\mathcal{G})$ For sets $X \subseteq \mathcal{L}_t$, $X\mathcal{G}\psi$ iff for every sequence $\vec{\phi}$ there is exactly one α such that $\vec{\phi}_{\alpha+1} \in X$, and nothing else is in X.

Theorem 10. For every $\phi \in \Phi_{lf}$ $\mathcal{L}\mathcal{G}\phi$.

Proof. By the definition of Δ_l, ϕ does not stand in the relation Δ_l to any ψ only if $\phi \in \Phi_1$. Hence, a sequence $\vec{\phi}$ terminates only in sentences of the first stage $\Phi_1 = \mathcal{L}$. Thus, the claim follows directly from lemma 8. $\square$

Theorem 11. $\mathcal{G}_l$ obeys Cut and Non-Circularity.

Proof. Firstly, $\mathcal{G}_l$ obeys Cut since if every ϕ_i is an element of a sequence $\vec{\psi}$ ($\{\phi_i : 1 \leqslant i \leqslant n\}\mathcal{G}_l\psi$) and every X_i collects exactly one χ from every $\vec{\phi_i}$ ($X_i\mathcal{G}_l\phi_i$), then $\bigcup\limits_{1 \leqslant i \leqslant n} X_i$ itself contains exactly one element from every sequence $\vec{\psi}$.

[7] For details of Yablo's work see appendix 7.3.

Secondly, $\mathcal{G}_l$ obeys Non-Circularity: whenever $X\mathcal{G}_l\phi$ then $X \neq \{\phi\}$. On the one hand, definition 9 requires every $\psi \in X$ to be a $\vec{\phi}_{\alpha+1}$, that is, not to be the first element of the sequence $\vec{\phi}$. On the other hand, by definition 7 every sentence ϕ stands in the relation Δ_l only to sentences of lower rank, and is therefore ensured not to stand in Δ_l to itself. Hence, for no α, $\vec{\phi}_{\alpha+1} = \phi$. $\square$

Thus, I have shown Leitgeb's definition of groundedness to satisfy the desideratum from the previous section. Every sentence grounded according to Leitgeb's definition stands in the relation $\mathcal{G}_l$ which obeys the rules of full, strict grounding.

Note that nothing of this definition hangs on features specific to Leitgeb's construction. Analogous relations can be defined for any definition of groundedness in terms of a monotone operator. In particular, there are such grounding relations for Kripke's Strong Kleene as well as his supervaluationist fixed point.

6 Conclusion

As a response to paradox, the notion of grounded truth needs to be supplemented with a notion of philosophical explanation. I considered different philosophical interpretations of groundedness none of which made the grounds of ϕ account for ϕ. Therefore, I proposed to understand semantic grounding as a relation of *sui generis* philosophical explanation: metaphysical grounding.

I clarified my proposal: semantic grounding obeys the rules of Kit Fine's *logic of ground* [2011]. Then, I tested my proposal against the most recent case of grounded truth theory: Leitgeb's (2005). As section §4 has shown, his concept of semantic dependence cannot be understood as a grounding relation. However, on the basis of work by Stephen Yablo (1982), I could obtain a relation that grounds every sentence in Leitgeb's fixed point and obeys Fine's logic.

In his (2005, 178), Hannes Leitgeb poses the question whether his concept of semantic dependence can be related to Yablo's work. I have done just this. Moreover, my definition of a *grounding* relation $\mathcal{G}_l$ is easily generalized, and applies equally to Kripke's definition of groundedness. Thus, I have also addressed a question posed by Kit Fine.

> The question of how the [metaphysical; js] relation of ground might be defined within the Kripkean framework is of great general interest [...] . (Fine 2010, 13)

	Strict	Weak
Full	$<$	$\leqslant$
Partial	$\prec$	$\preceq$

Table 1: Fine's concepts of grounding.

7 Appendix

7.1 Fine's Logic of Ground

Above, I have focused on full strict grounding and omitted the rules Fine provides for the four other concepts. I could do so, because the full logic does not prove any more facts of full, strict grounding allows than can be derived using full strict grounding alone. In this appendix, I will provide the details of this conservativeness result.

Definition 12. Let the four concepts of grounding be denoted as in table 1

Definition 13. (Sequents) Fix some language $\mathcal{L}$, not containing the symbols $\{<, \leqslant, \prec, \preceq\}$. For $\mathcal{L}$-sentences ϕ, ψ and sets of $\mathcal{L}$-sentences X, let a *sequent* s be any of the following expressions:

$$X < \phi$$
$$X \leqslant \phi$$
$$\phi \prec \psi$$
$$\phi \preceq \psi$$

Definition 14. (Pure Logic of Grounding) For sequents s and sets of sequents S, call $X < \phi$ *derivable* from S within the *Pure Logic of Grounding* ('$S \vdash_{PLG} X < \phi$) if s can be derived from S by the following two rules:

Subsumption

$$\frac{X < \phi}{X \leqslant \phi} \qquad \frac{X \prec \phi}{X \preceq \phi}$$

$$\frac{X \cup \{\phi\} < \psi}{\{\phi\} \prec \psi} \qquad \frac{X \cup \{\phi\} \leqslant \psi}{\{\psi\} \preceq \phi}$$

$$\text{Cut}\ \frac{X_1 \leqslant \phi_1 \qquad \ldots \qquad X_n \leqslant \phi_n \qquad \{\phi_i : 1 \leqslant i \leqslant n\} \leqslant \psi}{\bigcup_{1 \leqslant i \leqslant n} X_i \leqslant \psi}$$

$$\text{Transitivity}\ \frac{\phi \leqslant \psi \qquad \psi \leqslant \chi}{\phi \leqslant \chi} \qquad \frac{\phi \leqslant \psi \qquad \psi \prec \chi}{\phi \prec \chi} \qquad \frac{\phi \prec \psi \qquad \psi \leqslant \chi}{\phi \prec \chi}$$

$$\text{Identity}\ \frac{}{\phi \leqslant \phi} \qquad \frac{\phi \prec \phi}{\bot}\ \text{Non-Circularity}$$

$$\text{Reverse Subsumption}\ \frac{\{\phi_i : 1 \leqslant i \leqslant n\} \leqslant \chi \qquad \phi_1 \prec \chi \qquad \ldots \qquad \phi_n \prec \chi}{\{\phi_i : 1 \leqslant i \leqslant n\} < \chi}$$

7.2 Conservativeness of the Logic of Full Strict Grounding

Definition 15. (Pure Logic of Full Strict Grounding) Let the sequent $X < \phi$ be derivable from the set of sequents S within the *Pure Logic of Full Strict Grounding* (‘$S \vdash_{FS} X < \phi$’) if $X < \phi$ can be derived from S by the following two rules:

$$\text{Cut}(<)\ \frac{X_1 < \phi_1 \qquad \ldots \qquad X_n < \phi_n \qquad \{\phi_i : 1 \leqslant i \leqslant n\} < \psi}{\bigcup_{1 \leqslant i \leqslant n} X_i < \psi}$$

$$\text{Non-Circularity}\ \frac{X \cup \{\phi\} < \phi}{\bot}$$

Lemma 16. Fine’s Pure Logic Logic of Ground (PLG) is a conservative extension of the Pure Logic of Full Strict Grounding. For any sequent $X < \phi$ and any set of such sequents S

$$S \vdash_{PLG} X < \phi \quad \Rightarrow \quad S \vdash_{FS} X < \phi$$

Proof. (Nested induction on the length of derivation)

Assume $S \vdash_{PLG} X < \phi$, such that there is a derivation $\mathcal{D}$ of $X < \phi$ from S. If $X < \phi \in S$, then of course $S \vdash_{FS} X < \phi$. At the induction step, observe that a sequent of full strict grounding such as $X < \phi$ can only be derived in PLG by the rule of Reverse Subsumption. Hence, we can assume that $X = \{\psi_i : 1 \leqslant i \leqslant n\}$ and that the final inference in the derivation of $X < \phi$ from S in PLG is

$$\text{Reverse Subsumption}\ \frac{\{\psi_i : 1 \leqslant i \leqslant n\} \leqslant \phi \qquad \psi_1 \prec \phi \qquad \ldots \qquad \psi_n \prec \phi}{\{\psi_i : 1 \leqslant i \leqslant n\} < \phi}$$

The goal now is to show that this application of Reverse Subsumption is redundant, in the following sense. Either the premisses have been obtained from $\{\psi_i : 1 \leqslant i \leqslant n\} < \phi$ itself, in which case the induction hypothesis allows us to infer that $S \vdash_{FS} \{\psi_i : 1 \leqslant i \leqslant n\} < \phi$, or from premises that allow to infer $\{\psi_i : 1 \leqslant i \leqslant n\} < \phi$ by Cut($<$). In a slogan, the goal is "Reverse-Subsumption-Elimination".

For each premiss, this is done by an induction on the length of the derivation. First, consider $\{\psi_i : 1 \leqslant i \leqslant n\} \leqslant \phi$. Since it cannot have been in S, it must have been inferred from S within PLG. For the base case, assume that $\{\psi_i : 1 \leqslant i \leqslant n\} \leqslant \phi$ has been inferred directly from some sequent in S. Since every sequent in S is an $<$ sequent, this can only have happend by Subsumption($<$ / $\leqslant$), hence $\{\psi_i : 1 \leqslant i \leqslant n\} < \phi \in S$, and $S \vdash_{FS} \{\psi : 1 \leqslant i \leqslant n\} < \phi$. Now assume there is a derivation of $\{\psi : 1 \leqslant i \leqslant n\} \leqslant \phi$ from S of length $n + 1$. Its last rule applied is either Subsumption($<$ / $\leqslant$), or Cut($\leqslant$ / $\leqslant$). In the first case, we can infer $S \vdash_{FS} \{\psi : 1 \leqslant i \leqslant n\} < \phi$ as before. In the second case, we know by the "inner" induction hypothesis, that all the premisses have been inferred from corresponding $<$-sequents, such that the $\vdash_{FS}$ rule Cut($<$ / $<$) allows us to derive $\{\psi : 1 \leqslant i \leqslant n\} < \phi$, and by the "outer" induction hypothesis we can infer $S \vdash_{FS} \{\psi : 1 \leqslant i \leqslant n\} < \phi$.

Second, consider any $\{\psi_i\} \prec \phi$. For the base case, assume that it has been inferred directly from some sequent in S, using the PLF rule Subsumption($<$ / $\prec$). As before, this provides $S \vdash_{FS} \{\psi : 1 \leqslant i \leqslant n\} < \phi$. At the induction step, assume that $\{\psi_i\} \prec \phi$ has been derived from S by $n+1$ steps, and that every partial strict grounding sequent in this derivation has been inferred from an $<$-sequent. If this has been done by $Sub(< / \prec)$, then we reason as in the base case. Otherwise, $\{\psi_i\} \prec \phi$ has been inferred by one of the PLG Transitivity rules. Since S contains only $<$-sequents, the $\preceq$ premisses have been obtained either from $\prec$-sequents, such that the inner induction hypothesis requires them to trace back to $<$-sequents; or from $\leqslant$-sequents, which, by the outer induction hypothesis, themselves are derived from full strict grounding sequents. Either case allows for a $\vdash_{FS}$-proof of $\{\psi : 1 \leqslant i \leqslant n\} < \phi$, as desired. □

7.3 Yablo's Theory of Dependence

Let $U \neq \varnothing$, $S \subseteq U$ and $J : \mathcal{P}(U) \mapsto \mathcal{P}(U)$ be monotone.

$$\begin{aligned} J^0 &= S \\ J^{\alpha+1}(S) &= J(J^\alpha)(S) \\ J^\alpha(S) &= \bigcup_{\beta<\alpha} (J^\beta(S)), \text{ for } \alpha \text{ limit.} \end{aligned}$$

Let $S^\star$ be the least fixed point of this sequence.

Definition 17. (rank) The rank of an $x \in S^\star$ is the least α such that $x \in J^\alpha(S)$.

$$|x|_{J(S)} = min\{\alpha : x \in J^\alpha(S)\}$$

Definition 18. (Yablo 1982 Def. 5) $\Delta \subseteq U \times U$ is an S-dependence relation just in case

1. if $x \in S$ then $\neg\exists y : x\Delta y$,
2. otherwise,
   a) if $\exists R : x \in J(R)$ then $x\Delta y$ just in case $y \in R$, for some $R : x \in J(R)$ and all y,
   b) otherwise $x\Delta x$.

Remark 19. The gist of Yablo's *dependence* concept of groundedness can already be formulated. An object x is *grounded in* S if there is an S-dependence relation Δ such that every sequence of objects $(x, y, z \ldots)$ where $x\Delta y$, $y\Delta z$ and so on, is finite. Intuitively: Groundedness is "having a leg to stand on".

Definition 20. (Δ-path) Given an S-dependence relation Δ, a Δ-path is a sequence of objects $\langle y_0, y_1 \ldots)$ where for every α, $y_\alpha \Delta y_{\alpha+1}$.

Let '$\vec{x}$' denote any Δ-path whose first element is x.

Definition 21. (Yablo 1982 p. 122) An object x is *grounded in* S if there is an S-dependence relation Δ such that every Δ-path $\vec{x}$ is finite.

Remark 22. The following dependence relation plays a prominent role in Yablo's machinery. Figuratively speaking, it traces downwards the inductive definition of $S^\star$. This is achieved by restricting our attention to the sets $J^\alpha(S)$, which are unique for any $x \in J^{\alpha+1}(S)$.

Definition 23. (Yablo 1982, p. 123) Δ_S is an S-dependence relation such that

1. if $x \in S^\star$ then $x \Delta y$ just in case $y \in J^\alpha(S)$ for $|x|_{J(S)} = \alpha + 1$,
2. Otherwise, $x \Delta y$ just in case $y \in R$ for arbitrary $R : x \in J(R)$.

Lemma 24. (Yablo 1982 Prop. 7) For any $x \in S^\star$, x is grounded in S

In section 5 above, I applied this general result of Yablo's to the case of Leitgeb's definition of groundedness. In doing so, I relied on the following

Lemma 25. Δ_l as defined in definition 7 is an $\varnothing$-dependence relation in the sense of definitions 18 and 21.

Proof. First, recall that Leitgeb's operator D^{-1} is a monotone operator on sets of $\mathcal{L}_t$-sentences. His Φ_{lf} is a set $S^\star$, for $J = D^{-1}$.

Now, notice that for every ϕ of rank 1 , that is, every ϕ in Φ_1, Δ_l is undefined. Further, notice that $\phi \Delta_l \psi$ just in case $\phi \in \Phi_\alpha$ and $\psi \in \Phi_{\alpha+1}$, where, obviously, $D^-1(\Phi_\alpha) = \Phi_{\alpha+1} \ni \psi$. Hence, Δ_l is an $\varnothing$-dependence relation in the sense of definition 18

In fact, it also satisfies definition 21, since the rank function I defined for Leitgeb's inductive definition (p. 16) is an instance of the generic rank function $|\cdot|_{J(S)}$, in terms of which the relations Δ_S are defined. □

8 References

Audi, Paul. 2010. "A Clarification and Defense of the Notion of Grounding."

Correia, Fabrice. 2005. *Existential dependence and cognate notions.* Munich: Philosophia.

Daly, Christopher. 2011. "Scepticism about Grounding."

Field, Hartry. 2008. *Saving Truth from Paradox.* New York: Oxford University Press.

Fine, Kit. 2001. "The question of realism." *Philosophers' Imprint* 1: 1–30.

———. 2010. "Some Puzzles of Ground." *Notre Dame Journal of Formal Logic* 51: 97–118.

———. 2011. "The Pure Logic of Ground."

Halbach, Volker. 2011. *Axiomatic Theories of Truth.* Cambridge: Cambridge University Press.

Herzberger, Hans G. 1970. "Paradoxes of Grounding in Semantics." *Journal of Philosophy* 67: 145–167.

Hofweber, Thomas. 2009. "Ambitious, Yet Modest, Metaphysics." In *Metametaphysics: New Essays on the Foundations of Ontology*, 260–289.

Horsten, Leon. 2011. *The Tarskian Turn: Deflationism and Axiomatic Truth*. Cambridge, MA: MIT Press.

Kripke, Saul. 1975. "Outline of a Theory of Truth." *The Journal of Philosophy* 72 (November): 690–716.

Leitgeb, Hannes. 2005. "What Truth Depends On." *Journal of Philosophical Logic* 35: 155–192.

Maudlin, Tim. 2004. *Truth and Paradox: Solving the Riddles*. Oxford: Clarendon Press.

McCarthy, Timothy. 1988. "Ungroundedness in Classical Languages." *Journal of Philosophical Logic* 17: 61–74.

McGee, Vann. 1992. "Maximal Consistent Sets of Instances of Tarski's Schema (T)." *Journal of Philosophical Logic* 21: 235–241.

Rosen, Gideon. 2010. "Metaphysical Dependence: Grounding and Reduction." In *Modality: Metaphysics, Logic and Epistemology*, ed. Bob Hale and Aviv Hoffman. Oxford: Oxford University Press.

Yablo, Steve. 1982. "Grounding, Dependence and Paradox." *Journal of Philosophical Logic* 11: 117–137.